A Tensor SVD-based Classification Algorithm Applied to fMRI Data

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Katherine Keegan[1], Tanvi Vishwanath[2], and Yihua Xu[3]
Advised by Dr. Elizabeth Newman[4] and Vida John[5]

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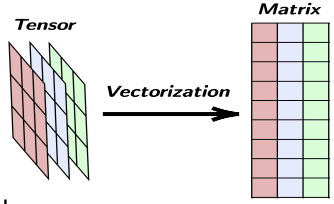
Abstract: To analyze the abundance of multidimensional data, tensor-based frameworks have been developed. Traditional matrix-based frameworks extract the most relevant features of vectorized data using the matrix-SVD. However, we may lose crucial high-dimensional relationships in this process. To facilitate efficient multidimensional feature extraction, we propose a projection-based classification algorithm using the t-SVDM, a tensor-based extension of the matrix-SVD. We apply our algorithm to the StarPlus fMRI dataset.

[1]Mary Baldwin University, Staunton, VA (keegank1624@marybaldwin.edu)
[2]Texas A&M University, College Station, TX (tanvivishwanath@tamu.edu)
[3]Georgia Institute of Technology, Atlanta, GA (yxu604@gatech.edu)
[4]Emory University, Atlanta, GA (elizabeth.newman@emory.edu)
[5]Tesselations School, Cupertino, CA

Matrix vs Tensor

Matrix Method:

- Uses matrix Singular Value Decomposition (SVD)
- Widely used in image processing
- Cannot identify relationships in higher dimensions



Tensor Method:

- Better representation of high-dimensional structure
- Flexibility in choosing a transformation matrix

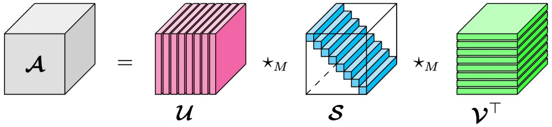
Background

- The mode - *k* product [5] refers to the multiplication of a matrix along the *k*-th dimension of the tensor.
- $\star_{\mathrm{M}}$ - product: Given tensors $\boldsymbol{\mathcal{A}} \in \mathbb{R}^{n_1\times n_2\times n_3}$, $\boldsymbol{\mathcal{B}} \in \mathbb{R}^{n_2\times \ell\times n_3}$, and an invertible $\boldsymbol{M} \in \mathbb{R}^{n_3\times n_3}$:

$$\boldsymbol{\mathcal{C}} = \boldsymbol{\mathcal{A}} \star_{\mathrm{M}} \boldsymbol{\mathcal{B}} = (\hat{\boldsymbol{\mathcal{A}}} \triangle \hat{\boldsymbol{\mathcal{B}}}) \times_3 \boldsymbol{M}^{-1}$$

where $\boldsymbol{\mathcal{C}} \in \mathbb{R}^{n_1\times \ell\times n_3}$ [3]

- Below displays the t-SVDM of a 3-D tensor [4]



Classification via Local t-SVDM [7]

Preprocessing

1. Split training data $\boldsymbol{\mathcal{A}}$ into c distinct classes:

$$\boldsymbol{\mathcal{A}}_1, \boldsymbol{\mathcal{A}}_2, \dots, \boldsymbol{\mathcal{A}}_c$$

2. For each class i, compute t-SVDM and store first k basis elements:

$$\boldsymbol{\mathcal{A}}_i = \boldsymbol{\mathcal{U}}_i \star_{\mathrm{M}} \boldsymbol{\mathcal{S}}_i \star_{\mathrm{M}} \boldsymbol{\mathcal{V}}_i^\top$$

$$\boldsymbol{\mathcal{U}}_{i,k} = \boldsymbol{\mathcal{U}}_i(:, 1:k, :)$$

Classifying a Test Image $\boldsymbol{\mathcal{T}}$

3. Project $\boldsymbol{\mathcal{T}}$ onto space spanned by each class basis:

$$\boldsymbol{\mathcal{P}}_i = \boldsymbol{\mathcal{U}}_{i,k} \star_{\mathrm{M}} \boldsymbol{\mathcal{U}}_{i,k}^\top \star_{\mathrm{M}} \boldsymbol{\mathcal{T}} \quad \text{for} \quad i = 1, \dots, c$$

4. Categorize $\boldsymbol{\mathcal{T}}$ as the class whose projection was "closest" to the original image:

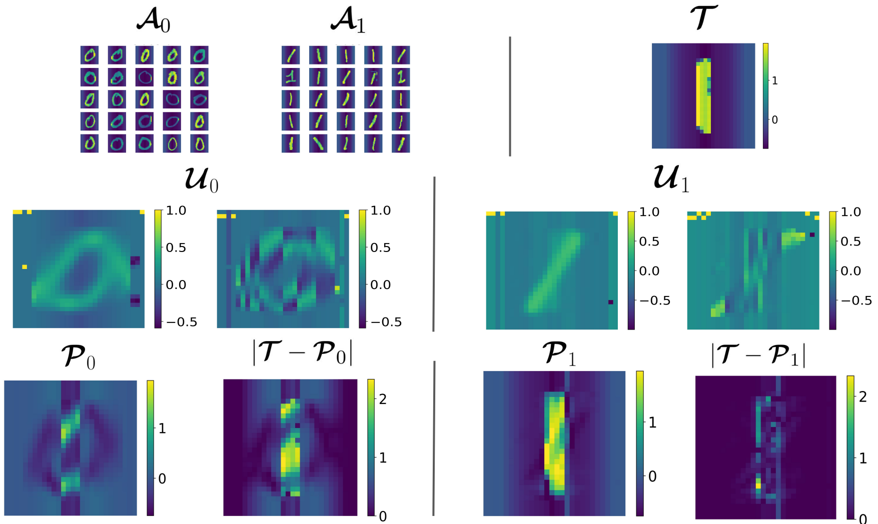
$$i^* = \operatorname*{arg\,min}_{i=1,\dots,c} \|\boldsymbol{\mathcal{T}} - \boldsymbol{\mathcal{P}}_i\|_F$$

5. To measure the performance of our algorithm,

$$\text{Accuracy} = \frac{\text{\# correctly classified images}}{\text{\# images}}$$

Intuition - MNIST [6]

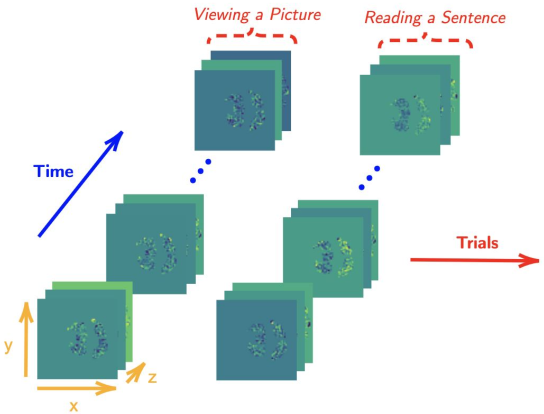
Below is an illustration of two classes $\mathcal{A}_0$ (representing digits consisting of 0) and $\mathcal{A}_1$(representing digits consisting of 1) go through the classification algorithm via local tensor SVD. Bases $\mathcal{U}_0$ and $\mathcal{U}_1$are generated by digits from class 0 and class 1, respectively. We project $\mathcal{T}$ onto the spaces spanned by and $\mathcal{U}_1$and obtain $\mathcal{P}_0$ and $\mathcal{P}_1$, respectively. Absolute difference images $|\mathcal{T}-\mathcal{P}_0|$ and $|\mathcal{T}-\mathcal{P}_1|$ are generated by the absolute pixel difference between $\mathcal{T}$ and $\mathcal{U}_0$, and $\mathcal{T}$ and $\mathcal{P}_1$.



Observations:

- $\mathcal{P}_0$ has characteristics of both digit 0 and digit 1
- $\mathcal{P}_1$ retains the characteristics of digit 1 only
- $\|\mathcal{T}-\mathcal{P}_0\|_F \approx 1.46 > \|\mathcal{T}-\mathcal{P}_1\|_F \approx 0.61$
- $\mathcal{T}$ classified as 1

StarPlus fMRI Data [2]



The StarPlus fMRI data consists of six human subjects completing 80 trials, each corresponding to the distinct cognitive tasks of viewing either a picture or a sentence.

Dimensions of Data

Tensor:

(trials, x, y, z, time) = (480, 64, 64, 8, 16)

Matrix (vectorized):

(x·y·z·time, trials) = (524288, 480)

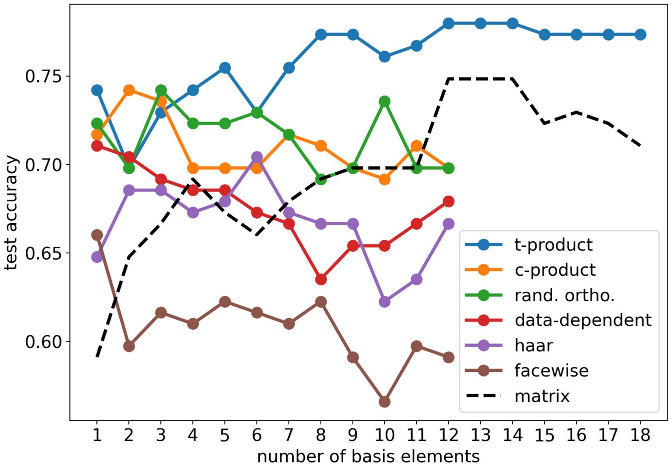
Power of Tensor Representations

Choices of M

Matrix	Advantage	Orthogonal
Banded	Temporal Dimension	No
Data-Dependent	Capture Structure Unique to Data	Yes
DCT	Separate Images with Varying Importance	Yes
DFT	Decompose Signals & Catch Spatial Shift	Yes
Haar	Capture Sudden Transitions of Signals	Yes
Identity	Benchmark	Yes
Random Orthogonal	Assume No Data Structure; Might Give Little Advantage over its Counterparts	Yes
ROI-Dependent	Understand Different ROIs' roles	Yes

Number of basis elements: Choose the k largest singular values and their corresponding tensors to be our basis element.

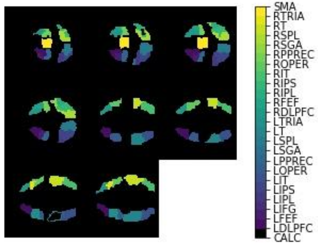
Results



Test accuracy with respect to number of basis elements for various choices of $\star_{\mathrm{M}}$ - product

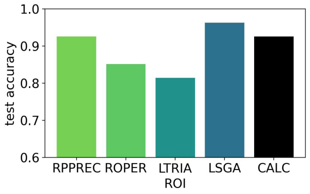
- Traditional matrix method overlooks the intrinsic characteristics of fMRI images as brain slices over time are very interconnected
- Tensor method outperforms matrix method in test accuracy with:
  - appropriate choice of transformation matrix
  - small number of basis elements

Impact of Brain Regions

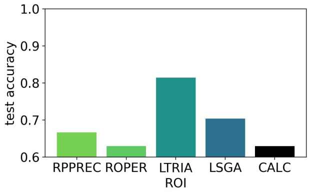


3D fMRI scan at single time point with twenty-five labeled Regions of Interest (ROI)

We also experiment with an ROI-dependent transformation matrix calculated from the most prominent ROI's in each trial.



Subject 1



Subject 2

Observations are the following:

- Best ROI's vary depending on the subject
- No specific regions consistently improve performance in all subjects
- Illustrates how humans complete these cognitive tasks differently, demonstrating the difficulty of creating a good universal basis

RPPREC = right posterior precentral sulcus, ROPER = right opercularis, LTRIA = left triangularis, LSGA = supramarginal gyrus, CALC = calcarine sulcus [1]

Conclusion

- Dimension Extension: The original t-SVDM framework is proposed for only three-dimensions. Our paper provides definitions for a *p*-dimensional tensor and illustrates the usability of this framework for a five-dimensional fMRI dataset.
- Transformation Choices: We select the t-SVDM not only for its ability to process high-dimensional data, but also for the flexibility that the framework introduces via the $\star_{\mathrm{M}}$-product, which enables one to strategically choose a mathematical transformation based on the nature of the data being analyzed.
- Algorithm Flexibility: The t-SVDM and our proposed classification procedure is a mathematically justified framework that can be applied to any labeled high-dimensional data. Thus, all of the methods described in our paper can easily be extended to other similar classification tasks with labeled data.

Future Work

- Store compressible basis
- Combine our method with other nonlinear classification methods, e.g. neural networks
- Find orientation-independent tensor approaches
- Explore applications that would potentially be useful for medical diagnostic classification tasks.

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